

饱和黏弹性地基土中管桩纵向振动研究

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摘 要: 用解析方法在频率域内研究考虑质量耦合效应的饱和黏弹性地基土中管桩的纵向振动特性。基于 Biot 理论, 采用薄层法, 推导得到饱和黏弹性地基土的位移、应力等的表达式。将管桩等效为一维弹性杆件处理。根据界面连续性条件, 给出饱和黏弹性地基土中管桩的纵向振动一般分析方法和桩顶动力复刚度的表达式。在该基础上, 对比分析饱和地基土中实心桩和管桩纵向振动特性。通过算例分析, 考察桩周土和桩芯土的力学参数对桩顶刚度因子和等效阻尼的影响。研究表明, 饱和黏弹性地基土中实心桩和管桩的纵向振动有明显的差异。

关 键 词: 饱和黏弹性土; 管桩; 纵向振动; 动力复刚度

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Study of vertical vibration of pipe pile in saturated viscoelastic soil

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Abstract: Using the analytical method, vertical vibration characteristics of pipe pile in saturated viscoelastic soil considering mass-coupling effects are investigated in the frequency domain. Based on Biot's dynamic theory, the expressions for displacement and stress of the saturated viscoelastic soil are obtained by using Novak's layer method. The pipe pile is treated as the Euler-Bernoulli rod. By utilizing the continuity conditions between the soil and pipe pile, a general analysis method for the vertical vibration of the pipe pile in the saturated viscoelastic soil is presented; and the expression of the complex dynamic stiffness at the pile top is obtained. On the basis, the comparative analysis between the vertical vibration characteristics of solid pile and one of pipe pile in saturated soil is made. By using the numerical example, the influences of mechanical parameters of soil around the pile and inner soil on the dynamic stiffness factor and equivalent damping at the pile top are investigated. It is shown that the vertical vibration characteristics of solid pile and pipe pile have significant difference.

Key words: saturated viscoelastic soil; pipe pile; vertical vibration; dynamic complex stiffness

1 引 言

从 20 世纪 20 年代以来, 管桩作为一种地基处理及桩基础的重要形式, 在软土地基、高速公路和港口码头等工程领域中得到广泛的应用^[1-3]。

近年来, 饱和土中桩振动特性研究开始备受众多学者关注, Maeso 等^[4]将桩和土分别视为弹性介质和流固两相介质, 采用三维边界单元模型研究了饱和半空间土中单桩和 2×2 群桩的动力刚度系数, 着重考察了激振频率、桩刚度和饱和土参数对桩顶复刚度的影响。王建华等^[5]将桩等效为圆柱梁单元,

在三维坐标下研究了饱和半空间土中群桩的动力响应, 分析了竖向力、孔压随桩距和桩刚度的分布规律。陆建飞等^[6]研究了简谐荷载作用下成层饱和半空间土中单桩的动力响应, 以 2 层土为例得到了动态刚度和等效阻尼随频率的变化曲线。李强等^[7-8]考虑三维波动效应, 采用分离变量法分别得到了饱和土中端承桩和弹性支撑桩纵向振动的解析解和半解析解, 讨论了桩土模量比、长径比、渗透性系数等参数对桩顶动态刚度和阻尼的影响。张智卿等^[9]利用三维模型, 研究了横观各向同性土中端承桩的扭转耦合振动特性。在此基础上, 王建华等^[10]考虑

孔隙水的流动特征,研究了饱和横观各向同性土中桩的扭转振动。杨晓等^[11]将桩等效为三维弹性介质,采用多孔介质理论在频率域内研究了饱和黏弹性土层中端承桩的纵向振动特性,考察了饱和土和桩力学参数对桩头复刚度的影响规律。杨晓等^[12-13]考虑端承桩的横向效应,将桩等效为黏弹性介质和 Rayleigh-Love 杆模型,分别研究了单相和饱和黏弹性土中单桩的纵向耦合振动特性。

饱和土中管桩的振动耦合理论不仅具有重要的学术价值,而且能为管桩的动力检测和抗震设计提供重要的理论支撑。然而,由于管桩的桩芯土和桩周土的力学参数不一致和数学模型的建立存在诸多困难,导致国内外学者对管桩动力特性的研究相对较少^[14]。

2 饱和黏弹性土层的动力响应

如图 1 所示,在厚度为 H 的饱和黏弹性土层中有一内外径分别为 R_1 和 R_2 的弹性端承圆柱管桩,其弹性模型和体密度分别为 E_p 和 ρ_p ,截面面积为 A_p ,桩顶 ($z=0$) 作用一圆频率为 ω 的简谐激振力 $p(t)=Pe^{i\omega t}$ ($i^2=-1$ 为虚数单位); $f_1(z)$ 、 $f_2(z)$ 分别为管桩的外侧和内侧摩擦阻力。假定管桩和饱和黏弹性土在纵向振动中均为小变形,且桩土完全紧密接触,无相对滑移,即满足应力、位移连续。

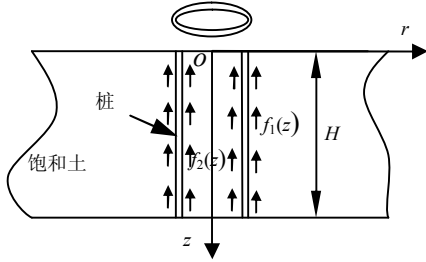


图 1 饱和黏弹性土中管桩计算模型

Fig.1 Calculate model of pipe pile in saturated viscoelastic soil

根据 Biot 波动原理^[15],饱和黏弹性土层的变形满足 Novak 薄层法的假定,即 u_z^S 、 w_z^F 物理量与纵向坐标 z 无关,则在动力荷载作用下土体的动力方程为^[16]

$$G^S(1+2\xi_1^S i)\left(\frac{\partial^2 u_z^S}{\partial r^2} + \frac{1}{r} \frac{\partial u_z^S}{\partial r}\right) = \quad (1)$$

$$\begin{aligned} & b \frac{\partial}{\partial t} (u_z^S - w_z^F) + \rho_v \frac{\partial^2 u_z^S}{\partial t^2} \\ & b \frac{\partial}{\partial t} (w_z^F - u_z^S) + \rho_w \frac{\partial^2 w_z^F}{\partial t^2} = 0 \end{aligned} \quad (2)$$

式中: $b = n^2 \eta_0 / k_d$ 为流固相互作用系数, η_0 为孔隙流体黏滞系数, k_d 为动力渗透系数; u_z^S 、 w_z^F 分别为固体和流相的竖向位移; $\rho_v = (1-n)\rho^S + \rho_a$; $\rho_w = n\rho^F + \rho_a$; ρ^S 、 ρ^F 分别为土骨架和孔隙流体的材料密度; ρ_a 为附件质量密度; G^S 、 ξ_1^S 分别为土体的剪切模量和阻尼比。

设桩周土和桩芯土的物理参数分别为 G_1^S 、 ξ_1^S 、 b_1 、 ρ_{v1} 、 ρ_{w1} 和 G_2^S 、 ξ_2^S 、 b_2 、 ρ_{v2} 、 ρ_{w2} , 两者的竖向位移分别 u_{z1}^S 、 w_{z1}^F 和 u_{z2}^S 、 w_{z2}^F , 由式(1)、(2)可得桩周土和桩芯土的动力方程:

$$\left. \begin{aligned} G_i^S(1+2\xi_i^S i)\left(\frac{\partial^2 u_{zi}^S}{\partial r^2} + \frac{1}{r} \frac{\partial u_{zi}^S}{\partial r}\right) &= \\ b_i \frac{\partial}{\partial t} (u_{zi}^S - w_{zi}^F) + \rho_{vi} \frac{\partial^2 u_{zi}^S}{\partial t^2} & \\ b_i \frac{\partial}{\partial t} (w_{zi}^F - u_{zi}^S) + \rho_{wi} \frac{\partial^2 w_{zi}^F}{\partial t^2} &= 0 \end{aligned} \right\} \quad (3)$$

对于稳态振动, 设 $u_{zi}^S = R_i U_{zi}^S e^{i\omega t}$, $w_{zi}^F = R_i W_{zi}^F e^{i\omega t}$ ($i=1,2$), 引入无量纲量和常数:

$$\left. \begin{aligned} G &= G_2^S / G_1^S; b_{21} = b_2 / b_1 \\ \rho_{v21} &= \rho_{v2} / \rho_{v1}; \rho_{w21} = \rho_{w2} / \rho_{w1} \\ \xi_{21}^S &= \xi_2^S / \xi_1^S; \eta = r / R_1 \\ \delta &= z / R_1; \bar{b}_1 = R_1 b_1 / \sqrt{\rho_1^S G_1^S} \\ \lambda &= R_1 \omega / V_1^S; V_1^S = \sqrt{G_1^S / \rho_1^S}; \rho_1^{FS} = \rho_1^F / \rho_1^S \end{aligned} \right\} \quad (4)$$

将式(4)代入式(3), 可得

$$\left. \begin{aligned} (1+2\xi_1^S i)\left(\frac{d^2 U_{z1}^S}{d\eta^2} + \frac{1}{\eta} \frac{dU_{z1}^S}{d\eta}\right) &= \\ \bar{b}_1 i \lambda (U_{z1}^S - W_{z1}^F) - \bar{\rho}_{v1} \lambda^2 U_{z1}^S & \\ \bar{b}_1 i \lambda (W_{z1}^F - U_{z1}^S) - \bar{\rho}_{w1} \lambda^2 W_{z1}^F &= 0 \end{aligned} \right\} \quad (5)$$

和

$$\left. \begin{aligned} G(1+2\xi_{21}^S i)\left(\frac{d^2 U_{z2}^S}{d\eta^2} + \frac{1}{\eta} \frac{dU_{z2}^S}{d\eta}\right) &= \\ b_{21} \bar{b}_1 i \lambda (U_{z2}^S - W_{z2}^F) - \rho_{v21} \bar{\rho}_{v1} \lambda^2 U_{z2}^S & \\ b_{21} \bar{b}_1 i \lambda (W_{z2}^F - U_{z2}^S) - \rho_{w21} \bar{\rho}_{w1} \lambda^2 W_{z2}^F &= 0 \end{aligned} \right\} \quad (6)$$

将式(5)中第2式代入第1式, 可得

$$\frac{d^2 U_{z1}^S}{d\eta^2} + \frac{1}{\eta} \frac{dU_{z1}^S}{d\eta} - q_1^2 U_{z1}^S = 0 \quad (7)$$

式中:

$$q_1^2 = \frac{1}{1+2\xi_1^S i} \left[\bar{b}_1 i \lambda + \frac{(\bar{b}_1)^2 \lambda^2}{\bar{b}_1 i \lambda - \bar{\rho}_{w1} \lambda^2} - \bar{\rho}_{v1} \lambda^2 \right] \quad (8)$$

式(7)可易解得

$$U_{z1}^S = A_1 K_0(q_1 \eta) + B_1 I_0(q_1 \eta) \quad (9)$$

根据贝塞尔函数的性质, 当 r 很大时, 有

$$\left. \begin{aligned} I_0(qr) &\sim (2\pi qr)^{-1/2} \exp(qr) \\ K_0(qr) &\sim (\pi qr/2)^{1/2} \exp(-qr) \end{aligned} \right\} \quad (10)$$

可知 $B_1 = 0$, 则

$$U_{z1}^S = A_1 K_0(q_1 \eta) \quad (11)$$

将式(6)中第2式代入第1式, 可得

$$\frac{d^2 U_{z2}^S}{d\eta^2} + \frac{1}{\eta} \frac{dU_{z2}^S}{d\eta} - q_2^2 U_{z2}^S = 0 \quad (12)$$

式中:

$$q_2^2 = \frac{1}{G(1 + 2\xi_{21}^S \xi_{11}^S i)} \left[b_{21} \bar{b}_1 i \lambda + \frac{(b_{21})^2 (\bar{b}_1)^2 \lambda^2}{b_{21} \bar{b}_1 i \lambda - \rho_{w21} \bar{\rho}_{w1} \lambda^2} - \rho_{v21} \bar{\rho}_{v1} \lambda^2 \right] \quad (13)$$

式(12)易解得

$$U_{z2}^S = A_2 K_0(q_2 \eta) + B_2 I_0(q_2 \eta) \quad (14)$$

再由桩芯土的性质 ($r \rightarrow 0, u_{z2}^S = 0$) 可知, $A_2 = 0$, 即

$$U_{z2}^S = B_2 I_0(q_2 \eta) \quad (15)$$

于是, 桩周土和桩芯土在桩土界面处的无量纲剪应力为

$$\bar{\tau}_{rz1} = -(1 + 2\xi_{11}^S i) A_1 q_1 K_1 \left(q_1 \frac{R_2}{R_1} \right) e^{i\omega t} \quad (16)$$

$$\bar{\tau}_{rz2} = G(1 + 2\xi_{21}^S \xi_{11}^S i) B_2 q_2 I_1(q_2) e^{i\omega t} \quad (17)$$

3 饱和黏弹性土中管桩的动力方程

设圆柱等截面管桩的弹性模量为 E_p , 其截面面积为 A_p , 在竖向荷载作用下的运动方程为

$$E_p A_p \frac{\partial^2 u_p}{\partial z^2} + 2\pi R_2 f_1(z) - 2\pi R_1 f_2(z) = \rho_p A_p \frac{\partial^2 u_p}{\partial t^2} \quad (18)$$

式中: $A_p = \pi(R_2^2 - R_1^2)$; $\tau_{rz1} = f_1(z)$; $\tau_{rz2} = f_2(z)$ 。

对于稳态振动, 设 $u_p = R_1 U_p e^{i\omega t}$, 将式(18)

无量纲化后, 可得

$$\begin{aligned} \bar{E}_p \left[\left(\frac{R_2}{R_1} \right)^2 - 1 \right] \frac{d^2 U_p}{d\delta^2} - \mathcal{G}_1 A_1 - \mathcal{G}_2 B_2 = \\ - \bar{\rho}_p \left[\left(\frac{R_2}{R_1} \right)^2 - 1 \right] \lambda^2 U_p \end{aligned} \quad (19)$$

式中:

$$\left. \begin{aligned} \mathcal{G}_1 &= 2 \frac{R_2}{R_1} (1 + 2\xi_{11}^S i) q_1 K_1 \left(q_1 \frac{R_2}{R_1} \right) \\ \mathcal{G}_2 &= 2 \frac{R_2}{R_1} (1 + 2\xi_{11}^S i) q_1 K_1 \left(q_1 \frac{R_2}{R_1} \right) \\ \bar{E}_p &= E_p / G_1^S; \bar{\rho}_p = \rho_p / \rho_1^S \\ \lambda &= R_1 \omega / V_1^S; V_1^S = \sqrt{G_1^S / \rho_1^S}; \delta = z / R_1 \end{aligned} \right\} \quad (20)$$

由桩土连续性条件, 可得

$$u_p \Big|_{r=R_2} = u_{z1}^S \Big|_{r=R_2}; u_p \Big|_{r=R_1} = u_{z2}^S \Big|_{r=R_1} \quad (21)$$

将式(18)代入式(17)可得

$$\frac{d^2 U_p}{d\delta^2} - \beta^2 U_p = 0 \quad (22)$$

式中:

$$\left. \begin{aligned} \beta^2 &= \frac{2}{\bar{E}_p \left[(R_2/R_1)^2 - 1 \right]} \left(\mathcal{G}_3 + \mathcal{G}_4 \right) - \frac{\bar{\rho}_p \lambda^2}{\bar{E}_p} \\ \mathcal{G}_3 &= \frac{R_2}{R_1} (1 + 2\xi_{11}^S i) q_1 \frac{K_1(q_1 R_2/R_1)}{K_0(q_1 R_2/R_1)} \\ \mathcal{G}_4 &= G(1 + 2\xi_{21}^S \xi_{11}^S i) q_2 \frac{I_1(q_2)}{I_0(q_2)} \end{aligned} \right\} \quad (23)$$

于是, 式(22)易解得

$$U_p = C_5 e^{\beta \delta} + C_6 e^{-\beta \delta} \quad (24)$$

桩土满足如下边界条件:

$$u_p \Big|_{z=H} = 0, \quad \frac{\partial u_p}{\partial z} \Big|_{z=0} = - \frac{p(t) e^{i\omega t}}{E_p \pi (R_2^2 - R_1^2)} \quad (25)$$

将式(25)进行无量纲化后, 可得

$$U_p \Big|_{\theta=H/R_1} = 0, \quad \frac{dU_p}{d\delta} \Big|_{\delta=0} = - \frac{p^*}{\bar{E}_p} \quad (26)$$

式中: $\theta = H/R_1$ 为桩的长径比; $p^* = p(t)/G_1^S \pi \cdot (R_2^2 - R_1^2)$ 。

将式(24)代入式(26), 可得

$$\left. \begin{aligned} C_5 e^{\beta \theta} + C_6 e^{-\beta \theta} &= 0 \\ C_5 - C_6 &= - \frac{p^*}{\beta \bar{E}_p} \end{aligned} \right\} \quad (27)$$

由式(27)可易解得

$$\left. \begin{aligned} C_5 &= - \frac{e^{-2\beta \theta} p^*}{\bar{E}_p \beta (1 + e^{-2\beta \theta})} \\ C_6 &= \frac{p^*}{\bar{E}_p \beta (1 + e^{-2\beta \theta})} \end{aligned} \right\} \quad (28)$$

于是

$$U_p = - \frac{e^{-2\beta \theta} p^*}{\bar{E}_p \beta (1 + e^{-2\beta \theta})} e^{\beta \delta} + \frac{p^*}{\bar{E}_p \beta (1 + e^{-2\beta \theta})} e^{-\beta \delta} \quad (29)$$

定义桩顶复刚度为

$$k_d = \frac{p^*}{R_1 U_p(0)} = \frac{\pi p^* G_1^S (R_2^2 - R_1^2)}{R_1 U_p(0)} \quad (30)$$

因此, 无量纲动力复刚度为

$$\bar{k}_d = \frac{p^*}{U_p(0)} = \frac{\bar{E}_p \beta (1 + e^{-2\beta\theta})}{1 - e^{-2\beta\theta}} \quad (31)$$

4 算例与数值分析

为了考察动刚度随频率的变化, 定义桩顶的动态刚度因子 $\kappa = \text{Re}(\bar{k}_d)/k_0$ 和桩土系统的等效阻尼 $\xi = \text{Im}(\bar{k}_d)/\lambda$, 其中, k_0 为桩的静态刚度, 即 $k_0 = \lim_{\omega \rightarrow 0} \text{Re}(\bar{k}_d)$, λ 为无量纲频率。

取如下无量纲参数: $n_1 = n_2 = 0.4$, $\theta = 10$, $\bar{\rho}_p = 25$, $G = 1$, $b_{21} = 1$, $\bar{b}_1 = 10$, $\rho_1^{\text{FS}} = 0.5$, $\bar{\rho}_{a1} = 0.05$, $\xi_{21}^S = 1$, $\xi_1^S = 0.1$, $R_2/R_1 = 1.1$, $\bar{E}_p = 1000$, $\rho_{21}^S = 1$, $\rho_{a21} = 1$, $\rho_{21}^F = 1$ 。

首先, 将本文计算结果与文献[7]进行对比分析。文献[7]采用三维模型, 研究了饱和土中实心端承桩的竖向振动特性, 得到了饱和土和桩力学参数对桩顶复刚度的影响规律曲线。图2为动刚度因子 κ 和等效阻尼 ξ 随无量纲频率 λ 的影响曲线。由图可见, 两者的结果存在很大的差异, 本文桩顶动力复刚度的振幅明显大于文献[7]的解答, 而波长却小于文献[7]的结果。主要存在3个原因: (1) 本文采用 Novak 薄层法研究管桩的竖向振动, 而文献[7]利用三维模型计算实心桩的动力复刚度; (2) 本文考虑了土体的黏性效应, 而文献[7]未考虑; (3) 本文考虑了土体的附加质量密度, 而文献[7]未考虑该值的影响。

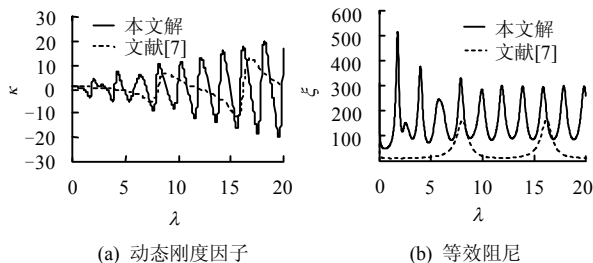


图2 本文结果与文献[7]结果对比分析

Fig.2 Comparison analysis for results between this paper and references [7]

图3为桩芯土和桩周土的剪切模量比对桩顶动力复刚度的影响曲线。由图可见, 随着模量比的增加, 即桩芯土较硬或桩周土较软时, 桩顶复刚度的

振幅有所减小, 而在低频处有所突变。这是由于桩芯土和桩周土的刚度变化引起的。

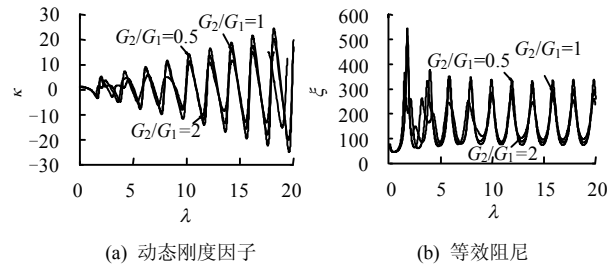


图3 剪切模量比对动力复刚度的影响

Fig.3 Influence of shear modulus ratio on dynamic complex stiffness

图4为桩芯土和桩周土流固相互作用系数的不同比值对桩顶动力复刚度的影响曲线。由图可见, 该系数对动力复刚度的影响很弱, 随着相互作用系数比值的增加, 动力复刚度随频率的变化曲线峰值呈减小趋势。该系数反映了土体的渗透性, 其值越大土体的渗透性降低。

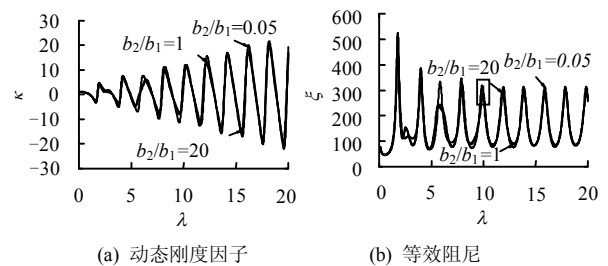


图4 流固相互作用系数对动力复刚度的影响

Fig.4 Influence of fluid-solid interaction coefficients on dynamic complex stiffness

图5为桩芯土和桩周土的不同阻尼比对桩顶复刚度的影响曲线。由图可见, 在低频处该比值对桩顶动态刚度因子和等效阻尼有较大影响。随着阻尼比的增加, 复刚度随频率变化曲线的峰值有所减小, 而在高频处该比值对桩顶复刚度影响很弱。

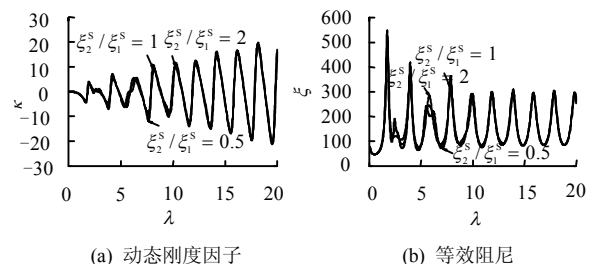


图5 土体的阻尼比对动力复刚度的影响

Fig.5 Influence of damping ratio on dynamic complex stiffness

图 6 为管桩的外半径和内半径比值对桩纵向振动的影响曲线。由图可见, 桩外半径越大或内半径越小, 桩顶复刚度随频率的变化曲线峰值越大, 即管桩的厚度越大, 峰值随之增大。这是由于管桩的厚度增加时, 桩的刚度也相应增大所引起的。图 7 为管桩长度和内半径的不同比值对桩顶动力复刚度的影响。由图可见, 随着长径比的增加, 刚度和阻尼振荡的幅度和波长明显减小, 同时, 桩顶复刚度产生的共振效应也相应减弱。这与桩基检测中振幅随桩长逐渐衰减的规律是一致的。图 8 为桩的弹性模量与桩周土的剪切模量比值变化时桩顶复刚度随频率变化的影响曲线。由图可见, 随着模量比的增加, 桩顶的动态刚度因子和等效阻尼的振幅和波长都明显增大, 产生的共振效应也明显增强。

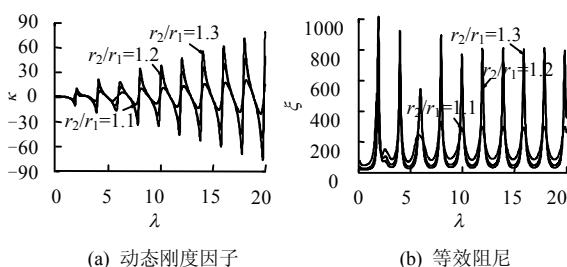


图 6 外内半径比对动力复刚度的影响

Fig.6 Influence of radius ratio in outer and inner on dynamic complex stiffness

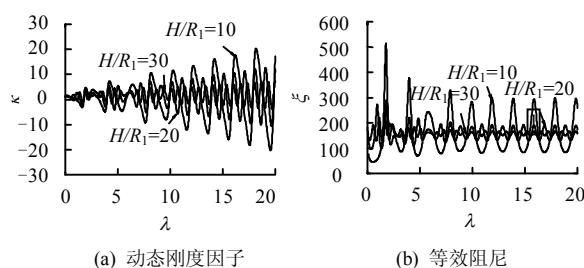


图 7 长径比对动力复刚度的影响

Fig.7 Influence of length-diameter ratio on dynamic complex stiffness

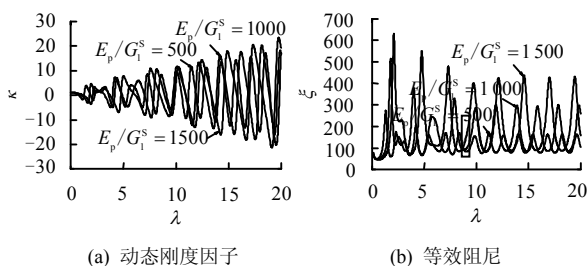


图 8 模量比对动力复刚度的影响

Fig.8 Influence of modulus ratio on dynamic complex stiffness

5 结 论

(1) 桩芯土和桩周土的剪切模量比、外内长径比对桩顶复刚度有很大影响, 而流固相互作用系数和阻尼比对管桩的纵向振动影响很弱。

(2) 管桩的桩顶复刚度随频率的变化曲线与实心桩有很大的差异。管桩的复刚度振幅明显要大于实心桩, 而波长相对较小。

(3) 随着管桩长度和内半径的比值的增加, 刚度和阻尼振荡的幅度和波长明显减小, 同时桩顶复刚度产生的共振效应也相应减弱。

(4) 随着桩的弹性模量与桩周土的剪切模量比值的增加, 桩顶的动态刚度因子和等效阻尼的振幅和波长都明显增大, 产生的共振效应也明显增强。

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